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Inflaton-induced sphaleron transitions

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Abstract: It has recently been proposed that the production of long wavelength Higgs and gauge configurations via parametric resonance at the end of inflation may give rise to the required baryon asymmetry at the electroweak scale. We show that the stability of the inflaton oscillations, long after the production of Higgs modes, keeps driving the sphaleron transitions, which then become strongly correlated to the inflaton oscillations. In models where the CP-violating operator is related to time variations of the Higgs field, these correlations immediately lead to an efficient generation of baryons that are not washed out after the resonance.

Keywords: Cosmology of Theories beyond the SM, Baryogenesis.

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Contents

1. Introduction

The origin of the matter-antimatter asymmetry in the universe is still one of the most pressing problems of cosmology. Until recently it was assumed that such an asymmetry could have arised at the electroweak scale through a first order phase transition [1, 2], or via leptogenesis [3] at a much higher temperature. Recently, a new mechanism for electroweak baryogenesis was proposed [4], based on the non-perturbative and out of equilibrium production of long-wavelength Higgs and gauge configurations via parametric resonance at the end of inflation. Such mechanism can be very efficient in producing the required sphaleron transitions that gave rise to the baryon asymmetry of the universe, in the presence of a CP-violating interaction.

The new scenario [4] considers a very economical extension of the symmetry breaking sector of the Standard Model with the only inclusion of a singlet scalar field σ that acts as an inflaton.[1] Its vacuum energy density drives a short period of expansion, diluting all particle species, and its coupling to the Higgs ϕ triggers the electroweak symmetry breaking. After inflation, the inflaton oscillations induce resonant Higgs production, via parametric resonance [5, 6], and out of equilibrium sphaleron transitions.

One of the major problems that afflicted previous scenarios of baryogenesis at the electroweak scale is the inevitability of a strong wash-out of the generated baryons after the end of the CP-violation stage during the phase transition. This problem was partially solved in the scenario of ref. [4], where CP violation and efficient topological (sphaleron) transitions coexist on roughly the same time scale, during the resonant stage of preheating, while after-resonance transitions are rapidly suppressed due to the decay of the Higgs and gauge bosons into fermions and their subsequent thermalization below 100 GeV.

[1] This field is not necessarily directly related to the inflaton field responsible for the temperature anisotropies of the microwave background.

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An important peculiarity of the new scenario is that it is possible for the inflaton condensate to remain essentially spatially homogeneous for many oscillation periods, even after the Higgs field has been produced over a wide spectrum of modes. These inflaton oscillations induce a coherent oscillation of the Higgs vacuum expectation value (VEV) through its coupling to the inflaton, and thus induce possible CP-violating interactions arising from operators containing the Higgs field. These oscillations affect the sphaleron transition rate Γ as well, since the Higgs VEV determines the height of the sphaleron barrier, therefore producing strong time correlations between variations in the rate Γ and the sign of CP violation.

We will show in this paper that such correlations can lead to counterintuitive dynamical effects responsible for the steady generation of baryons whenever sphaleron transitions occur, thus making the usual post-resonance wash-out unlikely, independently of the fermionic sector of the theory.

The key points are numerically illustrated with the $(1+1)$-dimensional abelian Higgs toy model, extended with a neutral (singlet) inflaton field, and a CP-violating operator [4]

$$\begin{aligned}\mathcal{L} = & -\frac{1}{4}F_{\mu\nu}^2 - \kappa|\phi|^2\,\epsilon_{\mu\nu}F^{\mu\nu} + \\ & + |D_\mu\phi|^2 - \frac{\lambda}{4}(|\phi|^2 - v^2)^2 + \\ & + \frac{1}{2}(\partial_\mu\sigma)^2 - \frac{1}{2}g^2\sigma^2|\phi|^2 \,. \end{aligned} \tag{1.1}$$

As described elsewhere [7, 8, 9, 10], this toy model contains all the necessary ingredients to study electroweak baryogenesis. Note, however, that we have not included the chiral fermions in the discussion.[2] We will assume for the moment that their coupling is weak enough that only after many inflaton oscillation the decay of the Higgs into fermions is relevant. The following discussion can be extended directly to the $(3+1)$-dimensional case.

It is also worth noticing that for a reasonably small $\kappa \lesssim 0.1$, the CP-violating term in (1.1) does not interfere with the dynamics of the inflaton and its ability to induce correlated sphaleron transitions. This means that our results are applicable to any field theory model with a $\phi^*\phi$-dependent CP-violating operator.

2. Baryogenesis after hybrid inflation

In the hybrid model of inflation considered in ref. [4], the effective Higgs mass vanishes at the end of inflation, due to its coupling to the inflaton field. This determines the inflaton amplitude at this moment:

$$g^2\sigma_c^2 = \lambda v^2 = M_H^2 \,, \tag{2.1}$$

[2]For a discussion of fermions in $(1+1)$ models, see ref. [11].

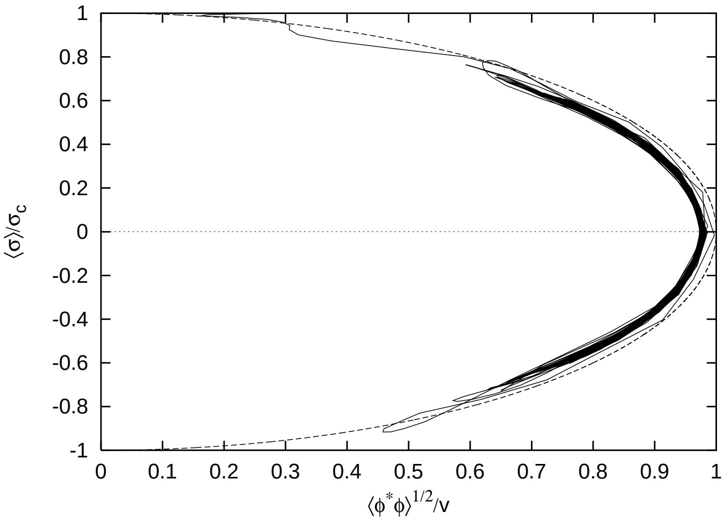

Figure 1: The inflaton and Higgs zero modes coherently oscillate near the trajectory $\sigma^2/\sigma_c^2 + \phi^*\phi/v^2 = 1$ (dashed line).

where M_H is the Higgs mass in the true vacuum, $\sigma = 0, \phi = v$. Due to the inflaton-Higgs coupling, the behaviour of these fields during preheating after inflation is non standard; see ref. [6]. The inflaton field is dominated by its homogeneous zero mode, which results in both Higgs and inflaton field coherently oscillating close to the trajectory along the minimum of the potential, $\sigma^2/\sigma_c^2 + \phi^*\phi/v^2 = 1$; see figure 1. This behaviour holds for many inflaton oscillations, even after the higher momentum modes of the Higgs field become populated via parametric resonance and rescattering. As a consequence, the effective Higgs mass and VEV are modified during that stage,

$$\tilde{v}^2 = v^2 - \frac{g^2}{\lambda}\langle\sigma^2\rangle = v^2\left(1 - \frac{\langle\sigma^2\rangle}{\sigma_c^2}\right) \tag{2.2}$$

$$\tilde{M}_H^2 = \lambda\tilde{v}^2 = M_H^2\left(1 - \frac{\langle\sigma^2\rangle}{\sigma_c^2}\right) \tag{2.3}$$

and the sphaleron mass, i.e. the height of the energy barrier, becomes (in $(1+1)$ dimensions, see refs. [7, 8])

$$E_{\rm sph} = \frac{4}{3}\sqrt{\lambda}\tilde{v}^3 = \frac{4}{3}\sqrt{\lambda}v^3\left(1 - \frac{\langle\sigma^2\rangle}{\sigma_c^2}\right)^{3/2}, \tag{2.4}$$

Note that eq. (2.4) holds only at the maxima and minima of $\langle\sigma^2\rangle$, when the Higgs potential is stationary.

Large coherent oscillations of the inflaton field naturally occur in the course of parametric resonance. Moreover, if the resonance doesn't result in a complete decay of the inflaton (see figure 2), in models without fermions, the inflaton keeps oscillating for a long period of time (figure 3), limited only by bosonic thermalization processes

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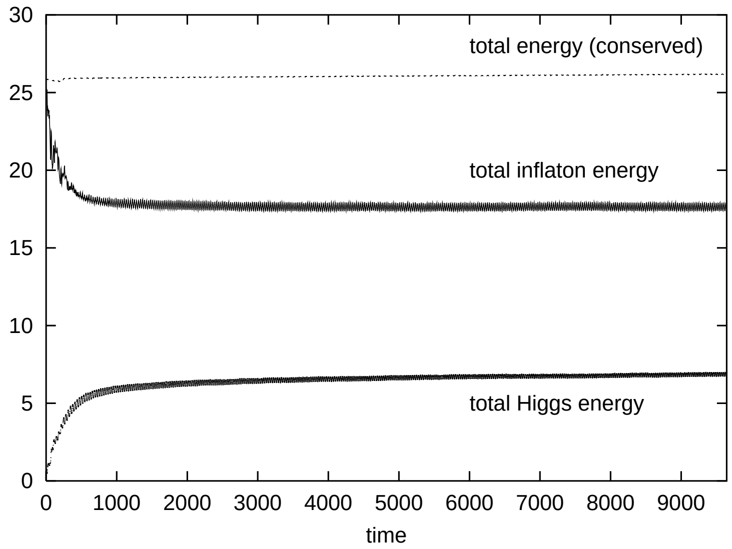

Figure 2: The time evolution of the inflaton and Higgs energies in the case of an incomplete resonance (in dimensionless units). The Higgs acquires here only $\sim 1/3$ of the initial energy, while the inflaton zero-momentum mode retains the remaining 2/3. For a detailed discussion of field spectra, units of measure and parameter values see ref. [12].

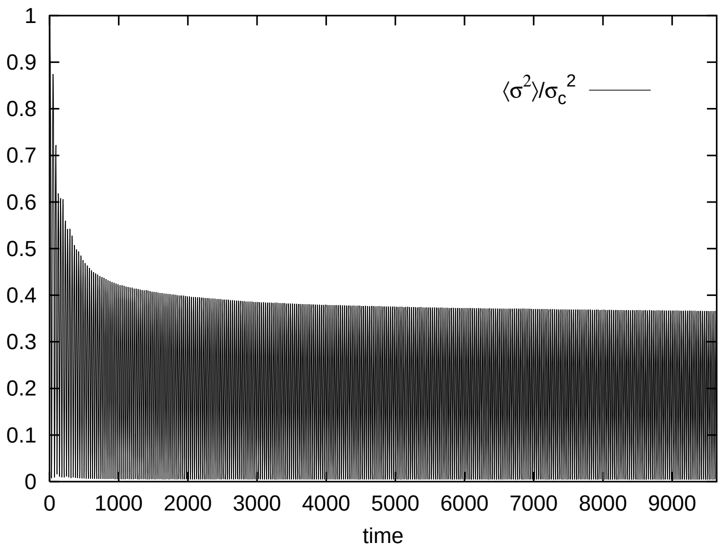

Figure 3: The time evolution of $\langle\sigma^2\rangle$. After the resonance ends (at $t \sim 1000$), the inflaton field stays almost completely homogeneous and keeps oscillating with an amplitude comparable to the initial one, $\sigma_c = 10$.

(which can be very slow [13, 14, 15, 16]) and the expansion of the Universe. As a consequence, one expects substantial periodic variations in the height of the energy barrier (2.4) separating different topological vacua. For stationary fields the barrier height equals the sphaleron mass (2.4) and its variations are close to a factor of $2^{3/2}$. In the low-temperature broken phase the topological transition rate exponentially

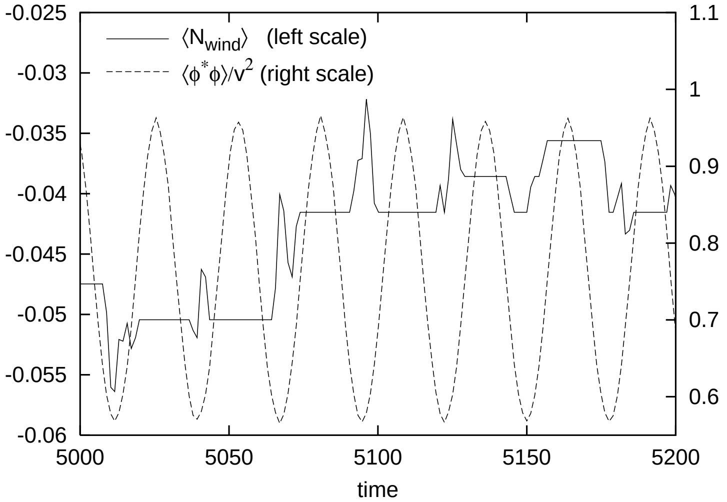

Figure 4: The periodic variations of the Higgs effective potential also affect the sphaleron transitions, detected by changes in the Higgs winding number, $N_{\rm wind}$. This figure shows the average over an ensemble of a few hundred runs (solid line). In our model no transitions occur when $\langle\phi^*\phi\rangle$ (dashed line) is close to its maximal value, which corresponds to the maximum height of the sphaleron barrier, $E_{\rm sph}$, see eq. (2.4), while the number of transitions is maximal when $\langle\phi^*\phi\rangle$ decreases to its minimum value. Of course, in each individual run $N_{\rm wind}$ has integer values.

depends on the barrier height, so these variations may result in large variations of the rate itself, especially if the exponential suppression factor is large.

Once $\langle\phi^*\phi\rangle$ keeps oscillating below v^2, even long after the beginning of the resonance, the periodical change in the barrier height, see eq. (2.4), will result in a corresponding increase of the sphaleron rate Γ, especially noticeable in the latter stages, when Γ becomes small, see ref. [4, Fig. 14]. Initially the transitions are not suppressed and hardly ever related to the barrier height. Note that the energy transfer from the inflaton to the Higgs in the course of parametric resonance will also result in a temporary increase in the rate Γ, mostly at the beginning of the resonance, see ref. [4].

3. Inflaton induced sphaleron transitions

An important consequence of the periodical variations in the barrier height is that the probability of topological transitions will change periodically in correlation with $\langle\phi^*\phi\rangle$ (see figure 4) and thus $\langle\sigma^2\rangle$ oscillations. This effect leads to interesting physical consequences for baryogenesis at preheating [4].

In most field theoretical models with CP violation due to a non-vanishing $\partial_0\langle\phi^*\phi\rangle$, such as the two-Higgs model [10] discussed in the context of baryogenesis at a thermal phase transition, the CP asymmetry is present, and baryons are generated, only for a short period of time, as the Higgs field moves to the new VEV through the phase

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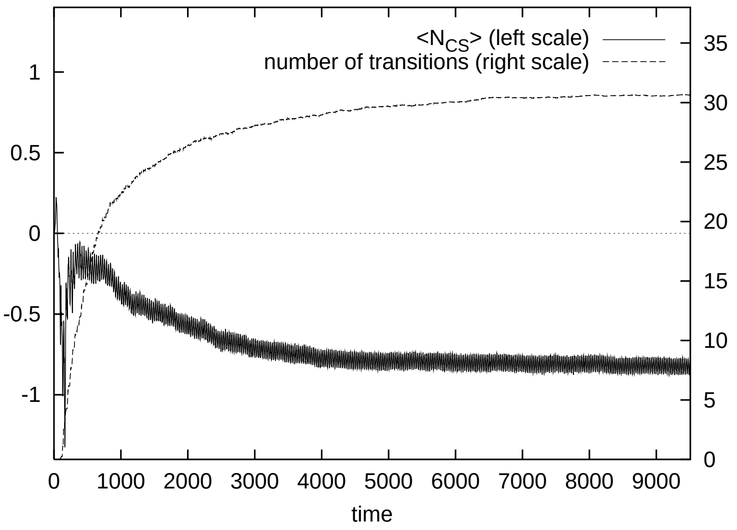

Figure 5: The continuous production of baryons as a result of correlations between the topological transition rate and the CP violation generated by the term $-\kappa\,\phi^*\phi\,\epsilon_{\mu\nu}F^{\mu\nu}$ in the lagrangean. This plot corresponds to $\kappa = 0.1$; for a detailed description see ref. [4, 12]. The solid line represents the shift in the Chern-Simons number, $N_{\rm CS} = \frac{1}{2\pi}\int A_1 dx$, averaged over an ensemble of a few hundred independent runs. The dashed line is the integral $\int \Gamma dt$, i.e. the average number of topological transitions accumulated per individual run. Note the remarkable similarity of both curves for $t > 1000$. This means that all transitions at this stage are equally efficient in generating baryons, changing the Chern-Simons number by about $-1/20$ per transition for many oscillations, demonstrating the absence of baryon wash-out in the model.

transition. Unfortunately, in those models the generated asymmetry could be rapidly washed out by late topological transitions.

On the other hand, in the recent proposal [4] for electroweak baryogenesis during preheating, the coherent long-term periodic oscillations of the inflaton naturally lead to a long-lived CP violation that shuts off only when the amplitude of oscillations finally decreases with thermalization. The oscillations in $\langle\phi^*\phi\rangle$ induce, via the CP-violating term in eq. (1.1), an alternating chemical potential $\mu_{\rm eff} \propto -\kappa\,\partial_0\langle\phi^*\phi\rangle$. This means that baryon and antibaryon production is biased in an alternating way with each oscillation. Depending on the phase of $\langle\phi^*\phi\rangle$ oscillations, CP violation changes sign and vanishes after time averaging. However, the topological transition rate is also changing in accordance with precisely the same phase, as described above, thus favouring CP violation of a certain sign — see figure 5 — that depends on the parameters and dynamics of the model; see figure 8 and ref. [12] for details.

It is this correlation between CP violation and the growth in the rate of sphaleron transitions which ensures that the baryonic asymmetry generated is completely safe from wash-out, because of the long-term nature of CP oscillations. Depending on initial conditions, the rate Γ can finally vanish, e.g. due to the (bosonic) thermalization of the Higgs field, as seen in figure 5, but this doesn't affect the continuous

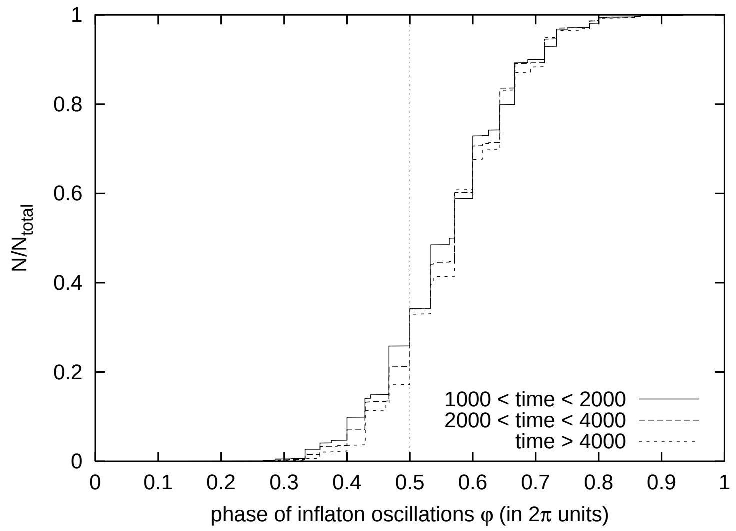

Figure 6: The distribution of sphaleron transitions as a function of the phase of inflaton oscillations (an integral histogram showing the ratio of transitions that happen at phases smaller than a given one). This figure shows that 75% of all observed transitions took place near the maximum of $\langle\sigma^2\rangle$, i.e. when the inflaton phase is in the range $0.4 < \varphi/2\pi < 0.7$, and therefore at the minimum of $\langle\phi^*\phi\rangle$. Note that the distribution is slightly asymmetric — more transitions happen after $\langle\phi^*\phi\rangle$ comes through its minimum (dotted line) — probably due to the delayed relaxation of the winding number to its new stable value. However, the exact origin of this slight time delay is yet to be understood.

pattern of CP-Γ correlations. In other words, these correlations effectively give rise to a permanent and constant CP violation, thus preventing the generated asymmetry from being washed out.

The non-equilibrium and non-perturbative nature of these inflaton-induced topological transitions makes their rigorous analysis difficult. However, it is possible to study these transitions numerically as in ref. [4], directly observing the time correlations between the transition rate Γ and the phase of $\langle\phi^*\phi\rangle$ oscillations by comparing the corresponding evolution plots, see figure 4. A more detailed analysis can be provided by the histogram distribution of the observed topological transitions with the phase of $\langle\sigma^2\rangle$ oscillations, figure 6. In this plot the phase $\varphi = 0$ or 2π corresponds to $\langle\sigma^2\rangle$ being at its minimum and $\langle\phi^*\phi\rangle$ at its maximum, and the phase $\varphi = \pi$ when the opposite is true. This figure clearly demonstrates that most transitions occur when $\langle\phi^*\phi\rangle$ is close to its minimum, where the sphaleron barrier is smaller. It also demonstrates that the $\Gamma - \langle\sigma^2\rangle$ correlation remains valid throughout the whole post-resonance period when (bosonic) thermalization of the Higgs results in a gradual decrease of the rate Γ.

On the other hand, in the case of complete resonance, when $\langle\sigma^2\rangle$ is small and the variations of $\langle\phi^*\phi\rangle$ have no significant effect on the rate, topological transitions become completely uncorrelated with the inflaton oscillations, see figure 7.

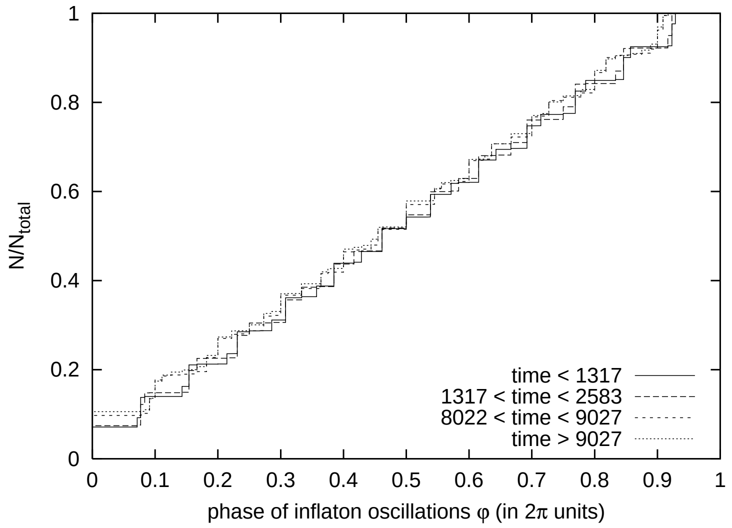

Figure 7: Analogous to figure 6 but for the case of a complete resonance; see ref. [4, Fig. 4]. This figure demonstrates the lack of correlations between Γ and $\langle\sigma^2\rangle$ oscillations in this case. The observed transitions are equally distributed with the phase φ.

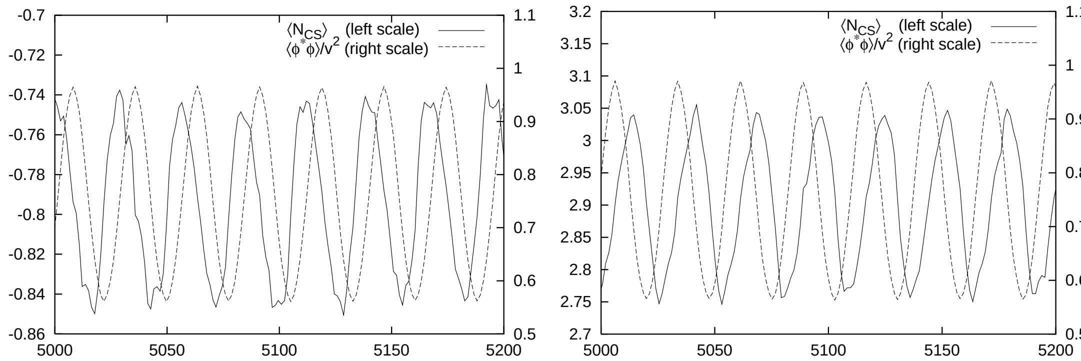

Figure 8: In these plots we show the relation between the periodical oscillations of Chern-Simons number N_{CS} driven by the CP-violating term and the Higgs oscillations that directly affect the sphaleron transitions (as shown on figure 4). The phase shift between $\langle N_{\text{CS}}\rangle$ and $\langle\phi^*\phi\rangle$ determines the sign of the permanent effective chemical potential which gives rise to a continuous production of antibaryons ($\kappa = 0.1$, left plot and figure 5), and baryons ($\kappa = -0.25$, right plot). Note that this phase difference exactly equals to $\frac{\pi}{2}\text{sign}\,\kappa$ and is insensitive to the absolute value of the CP-violation parameter.

4. Conclusions

It was shown in this paper that the presence of an oscillating inflaton field coupled to the Higgs can considerably modify the dynamics of both thermal and non-thermal topological transitions. Strong correlations between a $\phi^*\phi$-dependent CP-violating operator and the rate of sphaleron transitions Γ generate a permanent (long-lived) effective chemical potential driving baryogenesis. This extends the production of baryons for the whole period during which sphaleron transitions occur, therefore preventing the wash-out stage.

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